

۱. تبدیل لاپلاس توابع زیر را بدست آورید:

$$x(t) = \cos(5\pi t) \xrightarrow{\omega=5\pi} X(s) = \frac{s}{s^2 + (5\pi)^2} = \frac{s}{s^2 + 25\pi^2}$$

$$x(t) = t^3 u(t) \xrightarrow{n=3} X(s) = \frac{3!}{s^{3+1}} = \frac{6}{s^4}$$

$$x(t) = \sin\left(\pi t + \frac{\pi}{2}\right)$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\rightarrow x(t) = \sin(\pi t) \cdot \cos\frac{\pi}{2} + \cos\pi t \cdot \sin\frac{\pi}{2} = \cos(\pi t)$$

$$\rightarrow X(s) = \frac{s}{s^2 + \pi^2}$$

۲. معادلات دیفرانسیل زیر را به روش لاپلاس حل کنید:

$$1) \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} - 4y = 2u(t) \quad \rightarrow \text{صورت سوال اصلاح شده است}$$

$$y(0) = 0$$

$$\frac{dy(0)}{dt} = 0$$

$$\rightarrow \mathcal{L}\left\{\frac{d^2 y}{dt^2}\right\} + 3\mathcal{L}\left\{\frac{dy}{dt}\right\} - 4\mathcal{L}\{y\} = 2\mathcal{L}\{u(t)\}$$

$$\rightarrow (s^2 y(s) - sy(0) - y'(0)) + 3(sy(s) - y(0)) - 4y(s) = \frac{2}{s}$$

$$\rightarrow s^2 y(s) + 3sy(s) - 4y(s) = \frac{2}{s}$$

$$\rightarrow y(s) \underbrace{(s^2 + 3s - 4)}_{(s+4)(s-1)} = \frac{2}{s} \rightarrow y(s) = \frac{2}{s(s+4)(s-1)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s-1}$$

$$A = \lim_{s \rightarrow 0} s y(s) = \lim_{s \rightarrow 0} \frac{2}{(s+4)(s-1)} = -0.5$$

$$B = \lim_{s \rightarrow -4} (s+4) y(s) = \lim_{s \rightarrow -4} \frac{2}{3(s-1)} = \frac{2}{-4(-5)} = 0.1$$

$$C = \lim_{s \rightarrow 1} (s-1) y(s) = \lim_{s \rightarrow 1} \frac{2}{s(s+4)} = \frac{2}{5} = 0.4$$

$$\rightarrow y(t) = \mathcal{L}^{-1} \left\{ -\frac{0.5}{s} + \frac{0.1}{s+4} + \frac{0.4}{s-1} \right\}$$

$$= -0.5u(t) + 0.1e^{-4t}u(t) + 0.4e^{t}u(t)$$

$$2) \frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 18y = e^{-4t}u(t)$$

$$y(0) = 0$$

$$\frac{dy(0)}{dt} = 0$$

$$\mathcal{L} \left\{ \frac{d^2y}{dt^2} \right\} + 9\mathcal{L} \left\{ \frac{dy}{dt} \right\} + 18\mathcal{L} \{y\} = \mathcal{L} \{e^{-4t}u(t)\}$$

$$(s^2y(s) - sy(0) - y'(0)) + 9(sy(s) - y(0)) + 18y(s) = \frac{1}{s+4}$$

$$s^2y(s) + 9sy(s) + 18y(s) = \frac{1}{s+4}$$

$$y(s)(s^2 + 9s + 18) = \frac{1}{s+4} \rightarrow y(s) = \frac{1}{(s+3)(s+4)(s+6)} = \frac{A}{s+3} + \frac{B}{s+4} + \frac{C}{s+6}$$

$$A = \lim_{s \rightarrow -3} (s+3)y(s) = \lim_{s \rightarrow -3} \frac{1}{(s+4)(s+6)} = \frac{1}{3}$$

$$B = \lim_{s \rightarrow -4} (s+4)y(s) = \lim_{s \rightarrow -4} \frac{1}{(s+3)(s+6)} = -\frac{1}{2}$$

$$C = \lim_{s \rightarrow -6} (s+6)y(s) = \lim_{s \rightarrow -6} \frac{1}{(s+4)(s+3)} = \frac{1}{6}$$

$$\rightarrow y(t) = \mathcal{L}^{-1} \left\{ \frac{1/3}{s+3} - \frac{1/2}{s+4} + \frac{1/6}{s+6} \right\} = \frac{1}{3}e^{-3t}u(t) - \frac{1}{2}e^{-4t}u(t) + \frac{1}{6}e^{-6t}u(t)$$

۳. علی سبیل بیان نتایج زیر را می کنید.

$$1) H(s) = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2} \rightarrow n+1=2$$

$$\rightarrow n=1$$

$$h(t) = tu(t)$$

$$2) H(s) = \frac{s+1}{s^2 + 2s + 2} = \frac{s+1}{\underbrace{(s+1)^2 + 1}_{(s+1)^2 + \omega^2}} \rightarrow h(t) = e^{-t} \cos t u(t)$$

$\swarrow$   $\searrow$   
 $-a$   $\omega$

$$3) H(s) = \frac{s+2}{s^2+7s+12} = \frac{s+2}{(s+3)(s+4)} = \frac{A}{s+3} + \frac{B}{s+4}$$

$$A = \lim_{s \rightarrow -3} (s+3)H(s) = \lim_{s \rightarrow -3} \frac{s+2}{s+4} = -1$$

$$B = \lim_{s \rightarrow -4} (s+4)H(s) = \lim_{s \rightarrow -4} \frac{s+2}{s+3} = 2$$

$$\rightarrow h(t) = \mathcal{L}^{-1} \left\{ \frac{-1}{s+3} + \frac{2}{s+4} \right\} = -e^{-3t} u(t) + 2e^{-4t} u(t)$$

$$4) H(s) = \frac{5}{s^2+7s+10} = \frac{A}{s+2} + \frac{B}{s+5}$$

$$A = \lim_{s \rightarrow -2} (s+2)H(s) = \lim_{s \rightarrow -2} \frac{5}{s+5} = \frac{5}{3}$$

$$B = \lim_{s \rightarrow -5} (s+5)H(s) = \lim_{s \rightarrow -5} \frac{5}{s+2} = -\frac{5}{3}$$

$$\rightarrow h(t) = \mathcal{L}^{-1} \left\{ \frac{5/3}{s+2} - \frac{5/3}{s+5} \right\} = \frac{5}{3} e^{-2t} u(t) - \frac{5}{3} e^{-5t} u(t)$$

$$5) H(s) = \frac{2}{(s+2)^4}$$

$\swarrow$   $n+1=4$   
 $a$   $\rightarrow n=3$

$$\rightarrow H(s) = \frac{2 \times \frac{3!}{3!}}{(s+2)^4} = \frac{2}{3!} \cdot \frac{3!}{(s+2)^4} \rightarrow h(t) = \frac{1}{3} t^3 e^{-2t} u(t)$$

$\swarrow$   
 $3 \times 2$