

۱. الف) سمت حقیقی و موهومی اعداد مختلط زیر را بدست آورید: $+48i^2$

$$1) Z_1 = \frac{3-4i}{5+12i} \times \frac{5-12i}{5-12i} = \frac{15-36i-20i-48}{5^2+12^2} = \frac{-33-56i}{169} = -\frac{33}{169} - \frac{56}{169}i = -0.195 - 0.33i$$

$$2) (1+4i)^2 = 1^2 + 2 \times 1 \times 4i + (4i)^2 = (1-16) + 8i = -15 + 8i$$

$16i^2$

$$3) \underbrace{(1+2i)^3}_A + \underbrace{i(-1+i)}_B + \underbrace{(3+i)(4+5i)^2}_C = (-11-2i) + (-1-i) + (-67+111i) = -79 + 108i$$

$$A = (1+2i)^3 = 1^3 + 3 \times 1^2 \times (2i) + 3 \times 1 \times (2i)^2 + (2i)^3 = 1 + 6i - 12 - 8i = -11 - 2i$$

$$B = i(-1+i) = -i + i^2 = -1 - i$$

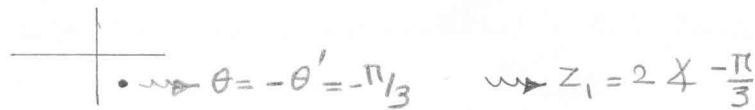
$$C = (3+i)(4+5i)^2 = (3+i)(16+40i-25) = (3+i)(-9+40i) = -27+120i-9i-40 = -67+111i$$

ب) اعداد مختلط زیر را بر حسب دگر مختلط تبدیل کنید:

$$1) Z_1 = 1 - \sqrt{3}i$$

$$r_1 = \sqrt{1^2 + (-\sqrt{3})^2} = 2$$

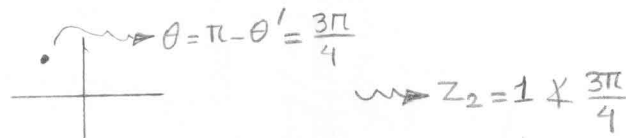
$$\theta'_1 = \tan^{-1} \left| \frac{\sqrt{3}}{1} \right| = \frac{\pi}{3}$$



$$2) Z_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

$$r_2 = \sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = 1$$

$$\theta'_2 = \tan^{-1} 1 = \pi/4$$



$$3) Z_3 = 2i$$



$$\Rightarrow Z_3 = 2 \times \frac{\pi}{2}$$

$$4) Z_4 = \sqrt{2} \angle 45^\circ = \sqrt{2} (\cos 45^\circ + i \sin 45^\circ) = \sqrt{2} \left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = 1 + i$$

$$5) Z_5 = \sqrt{3} \angle 60^\circ = \sqrt{3} (\cos 60^\circ + i \sin 60^\circ) = \sqrt{3} \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = \frac{\sqrt{3}}{2} + \frac{3}{2}i$$

$$Z_6 = 1 \angle 120^\circ = 1(\cos 120^\circ + i \sin 120^\circ) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

۲. نشان دهید:

$$1) \cos(a+b) = \frac{\cos a \cos b - \sin a \sin b}{I}$$

$$\begin{cases} \frac{e^{i\theta} + e^{-i\theta}}{2} = \cos \theta \\ \frac{e^{i\theta} - e^{-i\theta}}{2i} = \sin \theta \end{cases}$$

$$I = \left(\frac{e^{ia} + e^{-ia}}{2} \right) \left(\frac{e^{ib} + e^{-ib}}{2} \right) - \left(\frac{e^{ia} - e^{-ia}}{2i} \right) \left(\frac{e^{ib} - e^{-ib}}{2i} \right)$$

$$= \frac{1}{4} \left(e^{i(a+b)} + e^{i(a-b)} + e^{i(b-a)} + e^{-i(a+b)} \right) - \frac{1}{4i^2} \left(e^{i(a+b)} - e^{i(a-b)} - e^{i(b-a)} - e^{-i(a+b)} \right)$$

$$= \frac{1}{4} \left(e^{i(a+b)} + e^{i(a-b)} + e^{i(b-a)} + e^{-i(a+b)} + e^{i(a+b)} - e^{i(a-b)} - e^{i(b-a)} - e^{-i(a+b)} \right)$$

$$= \frac{1}{4} \left(2e^{i(a+b)} + 2e^{-i(a+b)} \right) = \frac{e^{i(a+b)} + e^{-i(a+b)}}{2} = \cos(a+b)$$

$$2) e^{-i\theta} = \cos \theta - i \sin \theta$$

$$I = \left(\frac{e^{i\theta} + e^{-i\theta}}{2} \right) - i \left(\frac{e^{i\theta} - e^{-i\theta}}{2i} \right) = \frac{1}{2} \left(e^{i\theta} + e^{-i\theta} - e^{i\theta} + e^{-i\theta} \right) = \frac{1}{2} (2e^{-i\theta}) = e^{-i\theta}$$

$$3) e^{i\theta} = \cos \theta + i \sin \theta$$

$$I = \left(\frac{e^{i\theta} + e^{-i\theta}}{2} \right) + i \left(\frac{e^{i\theta} - e^{-i\theta}}{2i} \right) = \frac{1}{2} \left(e^{i\theta} + e^{-i\theta} + e^{i\theta} - e^{-i\theta} \right) = \frac{1}{2} (2e^{i\theta}) = e^{i\theta}$$

۳. محسوس ها را در زیر را محاسبه کنید:

$$A_1 = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 5 & 8 \\ 2 & 4 & 0 \end{pmatrix}$$

$$|A_1| = \begin{vmatrix} 1 & -2 & 1 & | & 1 & -2 \\ 0 & 5 & 8 & | & 0 & 5 \\ 2 & 4 & 0 & | & 2 & 4 \end{vmatrix} = (-32) - (10 - 32) = -74$$

$$M_{11} = \begin{vmatrix} 5 & 8 \\ 4 & 0 \end{vmatrix} = -32$$

$$M_{12} = \begin{vmatrix} 0 & 8 \\ 2 & 0 \end{vmatrix} = -16$$

$$M_{13} = \begin{vmatrix} 0 & 5 \\ 2 & 4 \end{vmatrix} = -10$$

$$M_{21} = \begin{vmatrix} -2 & 1 \\ 4 & 0 \end{vmatrix} = -4$$

$$M_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} = -2$$

$$M_{23} = \begin{vmatrix} 1 & -2 \\ 2 & 4 \end{vmatrix} = 8$$

$$M_{31} = \begin{vmatrix} -2 & 1 \\ 5 & 8 \end{vmatrix} = -21$$

$$M_{32} = \begin{vmatrix} 1 & 1 \\ 0 & 8 \end{vmatrix} = 8$$

$$M_{33} = \begin{vmatrix} 1 & -2 \\ 0 & 5 \end{vmatrix} = 5$$

$$\Rightarrow A_1^{-1} = \frac{1}{(-74)} \begin{pmatrix} -32 & 16 & -10 \\ +4 & -2 & -8 \\ -21 & -8 & 10 \end{pmatrix}^T = \frac{1}{74} \begin{pmatrix} 32 & -4 & 21 \\ -16 & 2 & 8 \\ 10 & 8 & -5 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 1 & 7 \\ 2 & -2 & 2 \end{pmatrix} \xrightarrow{\times 2} \Rightarrow |A_2| = 0 \Rightarrow \text{معلوم نثری ہے}$$

$$B_1 = \begin{pmatrix} 11 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & -8 \end{pmatrix} \xrightarrow{\text{مقلی}} B_1^{-1} = \begin{pmatrix} \frac{1}{11} & 0 & 0 \\ 0 & \frac{1}{15} & 0 \\ 0 & 0 & -\frac{1}{8} \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 10 & 3 & 8 \\ 0 & 3 & 0 \\ -10 & 0 & -8 \end{pmatrix} \xrightarrow{\times \frac{10}{8}} \Rightarrow |B_2| = 0 \Rightarrow \text{معلوم نثری ہے}$$